POSTSCRIPT

to lectures on
Propagation and Acceleration of High-Energy Cosmic Rays

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After the lectures, questions were asked about the basic Fermi acceleration mechanism (which underlies diffusive shock acceleration), and about the origin of the factor 4 for the compression ratio. The plausible spectrum of extragalactic cosmic rays suggested at the end of the last lecture (page HIL-8 of the notes) deserves further comment, especially in relation to a somewhat different version put forward by Venya Berezinsky later in the week.

(i) The basic simplicity of Fermi's acceleration mechanism certainly was not made clear in the notes.

Consider just a single encounter of a charged particle with an approaching plasma cloud, inside which there is a magnetic field that is stationary with respect to the plasma, but which can cause the particle to turn back and re-emerge from the plasma.
According to Fermi, the particle in effect bounces from the advancing plasma mass. In the extreme relativistic case (v practically equal to c for the particle), the rest mass of the particle is negligible and the energy-momentum relationships are the same as for a photon. In the more familiar case of a photon bouncing from a mirror approaching at speed v, the photon recoils with a fractional energy (or frequency) gain 2 v/c, for a head-on collision (i.e. normal incidence). For the reflected cosmic-ray particle the only difference is that we average over all angles of incidence, assuming random isotropic motion, and the average fractional energy gain becomes 4/3 v/c. (For a non-relativistic particle the fractional momentum gain is 4/3 $v/v_{particle}$.) Note that the particle is reflected from the plasma somewhere behind the shock. It is not usually reflected from the shock itself (except in the case of magnetic field lines highly inclined to the shock normal).

Spectral form: Consider N particles entering the sequence of bounces from a plasma mass, with initial momentum p_1. If each bounce from the ``mirror'' produces a momentum change dp = a.p , i.e. dlnp = a,
and if the number, N, of particles still bouncing around changes by dN = -b.N: i.e. dlnN = -b at each bounce (some particles being lost from the system of acceleration – i.e. lost inside the SNR in the usual picture), the number of particles which experience at least r reflections is N = N_1 exp(-rb), and these particles reach a momentum of p = p_1 exp(ra) or more. Thus the number of particles reaching a momentum p or more is N = N_1 (p/p_1)-b/a : i.e. N is proportional to $p^{-b/a}$. The momentum distribution of these particles escaping from the acceleration process is thus, by differentiation, $|dN/dp| = C.p^{-\gamma}$, where $\gamma = 1 + b/a$.

In the collision with the SNR, the particle crosses the front, and moves about inside the compressed hot gas; and if it travels back out again -- which it does with probability (1 – v_{shock}/c) if the compression factor is 4 – it emerges with energy increased on average by a factor v_{shock}/c (for a relativistic particle). It has bounced from the high-speed advancing gas, and has been knocked back with increased momentum. If the compression ratio is 4, one finds that a fraction v_{shock}/c fail to return to the shock after each bounce. (Bell used a very simple argument taken from elementary kinetic theory of gases to show that the number of particles striking unit area of the shock front per second is (c/v_{shock}) times the number escaping downstream, so each particle has made on average c/v_{shock} crossings.)
So in this case, a = <dlnp> per crossing = v_{shock}/c, and b = v_{shock}/c, so the spectral exponent of particles accumulating inside the SNR is $\gamma = 1 + 1/1 = 2.0$ – so long as the compression ratio 4 holds.

(ii) Why the gas is compressed by a factor 4 if we neglect pressures other than pressure of the heated gas.

If a shock front moves at speed v_s into external gas of density ρ, a mass ρv_s is swept up per unit area per second.
If the gas is compressed by a factor σ, the compressed gas recedes from the shock front at a speed v_s/σ relative to the front, so its speed changes by $\Delta v = v_s(1 - 1/\sigma)$. Seen in the frame moving with the compressed gas, behind the shock, the gas has now no bulk momentum: it has lost momentum $\rho v_s \Delta v$ per second per unit area of the shock surface, and so the shocked gas must exert a pressure $p = \rho v_s \Delta v$ . . (eq.1). All the kinetic energy $\frac{1}{2}\rho v_s (\Delta v)^2$ swept up per unit area per second must now be random thermal energy (as the gas is stationary), so the thermal energy density in the compressed gas must be $\frac{1}{2}\rho\sigma (\Delta v)^2$, as the volume of this compressed gas added per unit area per second is now v_s/σ. The pressure of a non-relativistic non-molecular hot gas is 2/3 of the internal energy density . . (eq.2), so equating two expressions (eq.1 and eq.2) for the pressure, we have $1/3\, \rho\sigma (\Delta v)^2 = \rho v_s \Delta v$, giving $\sigma v_s(1 - 1/\sigma)/3 = v_s$,
and hence the density compression ratio $\sigma = 4$.

(iii) An equally simple alternative energy spectrum for extragalactic cosmic rays.

I showed a possible way in which different components of cosmic rays added together at the end of the notes (last page, HIL-8, of notes handed out).
There is an alternative version of the extragalactic component, advocated by Berezinsky in one of his talks.
If one does not require the same spectral exponent (about 2.36) for the extragalactic sources as the Galactic sources – and if they have a different geometrical shape and different development in time this may certainly be possible even for shock acceleration – it is possible to assume a pure proton component with a spectrum (at source) something like $E^{-2.5}$ to $E^{-2.6}$, I think, which could match the total observed cosmic ray flux from above 10^{19} eV right down to about 4×10^{17} eV, before turning over and becoming a lesser fraction of the total at lower energies. (The proton component which I showed as the line ``EGp'' had this same turn-over near 4×10^{17} eV, due to the steep dip at higher energies caused by pair-production reactions in space – but the new alternative has the whole curve tilted upwards at lower energies because of a source spectrum with a different exponent.) This alternative, then, is assumed to account for all cosmic rays above 4×10^{17} eV as extragalactic, rather than having a Galactic-extragalactic mixture below 10^{18} eV. It turns out that such a softer-spectrum alternative would match quite well to the highest energy Galactic (iron) component, and so this version also adds up well to give the observed total cosmic ray spectrum. Its merit is that it would not require a ``B component'' of the Galactic cosmic rays.

Thus, without a B component, it looks simpler. However, despite its artificial appearance, the B component had been taken to correspond to a component required by Bell's diffusive shock acceleration model – the SNRs expanding rapidly into a dense pre-supernova wind. But it may well be that Bell's model generates a smaller extended (B) component, so something in between the version drawn in the notes and this alternative may also be a possibility. *The test* will be to measure the composition of cosmic rays at around 4×10^{17} eV. The composition shown in the notes (HIL-8) would be only about 1/3 protons, and largely iron nuclei. The alternative version described here, and favoured by Berezinsky, would be almost entirely protons.
(Berezinsky's version is not exactly like the alternative I describe here, as he did not take source activity to be stronger in the past.)